

## Beam Splitters for Quantum Applications

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## Summary

This white paper provides an in-depth look at beam splitters, essential hardware for quantum technologies, with applications in quantum computing and quantum key distribution.

## Introduction

Beam splitters facilitate quantum information processing and secure communication protocols, playing an important role in implementing linear optical quantum gates, performing Bell state measurements, and enabling quantum key distribution (QKD). In classical optics, beam splitters manipulate light by splitting light waves into a reflected path and a transmitted path. In quantum optics, beam splitters can be thought of as a passive switch for photons. Beam splitters also can be used as a theoretical tool to model quantum optics processes such as quantum transduction.

## What does a beam splitter do?

At its core, a beam splitter is a device that performs specific operations on light. These operations have a wide range of applications, including:

**Quantum Computing.** In photonic quantum computing, beam splitters are used as quantum gates.

**Quantum Key Distribution (QKD).** In QKD protocols such as BBM92, beam splitters can act as passive switches, facilitating secure communication.

**Bell State Measurements.** Beam splitters can be used to perform Bell state measurements, which are used to generate and verify quantum entanglement. This capability is useful for quantum teleportation and entanglement-based communication protocols.

**Generalizable Model.** Beam splitters can be used to understand other quantum optics processes, including modeling quantum frequency conversion, or transduction.

As physical components, beam splitters come in two primary forms:

**Free-space beam splitters.** This includes plate beam splitters, pellicles, and cube beam splitters.

**Fiber-based beam splitters.** These operate within optical fibers and often incorporate integrated photonic circuits to perform operation on light.

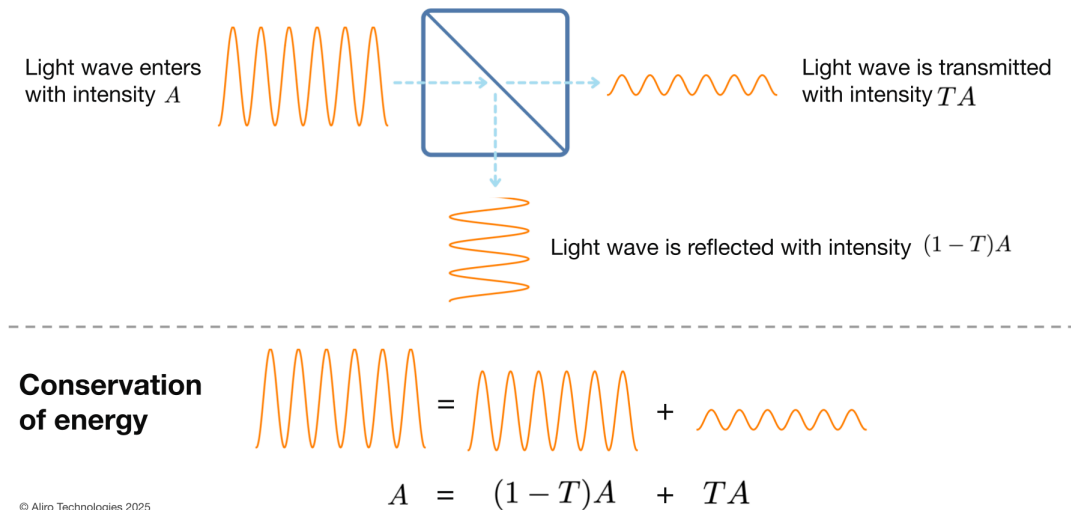
The choice between free-space and fiber-based beam splitters depends on the specific application, with free-space designs often used in laboratory experiments and fiber-based designs favored for integrated quantum systems.

## How beam splitters work (classical optics)

In classical optics, light behaves like a wave. When a light wave enters a beam splitter, some of the light is transmitted, and some of the light is reflected. Because of conservation of energy, the sum of the intensities of the transmitted light and reflected light will equal the intensity of the incoming light wave:  $A = (1 - T)A + TA$

### What is a beam splitter? (Classical picture)

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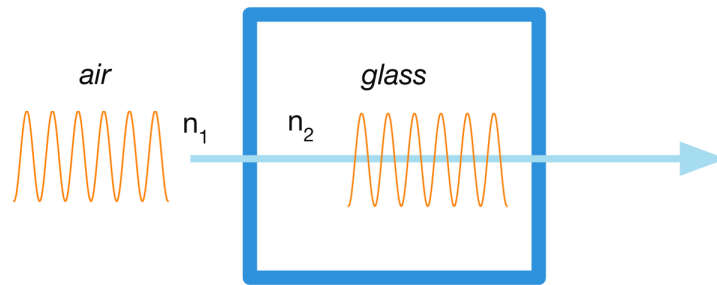


When a light wave exits a beam splitter it may undergo a phase shift, regardless of whether it is reflected or transmitted. A phase shift is a small delay that can cause interference effects.

The effect of a beam splitter on light can be analyzed using ray optics. When a ray of light enters a material, its behavior will be affected by the angle of incidence (the angle at which light enters the material) and the material's refractive index. Consider the simple case of light traveling from air with refractive index  $n_1$ , into glass with refractive index  $n_2$ .

If a ray of light enters from air into glass at a perpendicular angle of incidence, there is a right angle between the direction that the ray is traveling and the surface that it's entering into. The air has some index of refraction  $n_1$  and the glass has some index of refraction  $n_2$ . In this scenario, light passes straight through the material without deflecting at all, because the angle of refraction aligns with the angle of incidence.

## Reflection and refraction



A ray of light entering a material at a perpendicular angle of incidence will travel straight through.

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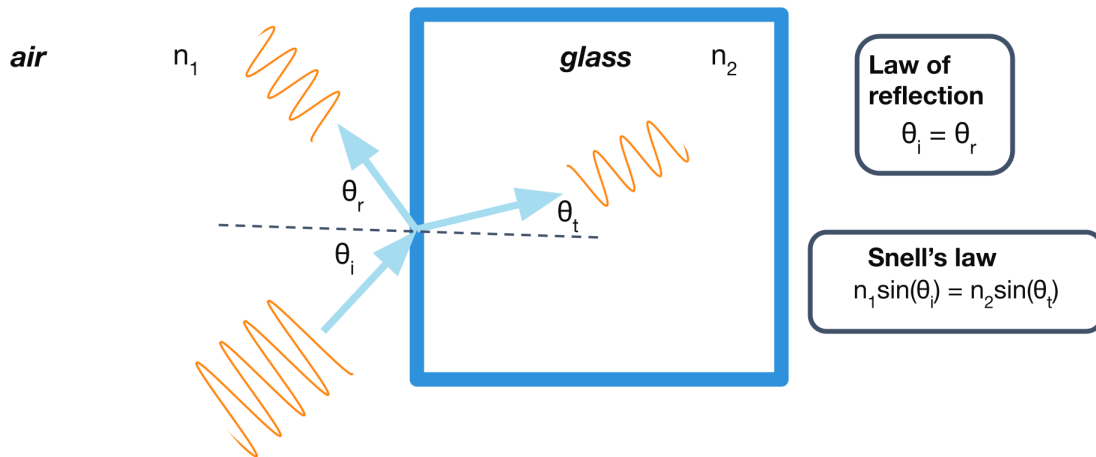
For a ray of light entering at an angle,  $\theta_i$ , the behavior becomes more complex. The ray splits into two components:

Transmitted (refracted) light,  $\theta_t$ . Transmitted light bends (refracts) as it enters the new material, as indicated by Snell's law.

Reflected light,  $\theta_r$ . Reflected light bounces off the surface of the material at an angle equal to the angle of incidence, as indicated by the law of reflection.

The law of reflection tells us that the angle of incidence will always equal the angle of reflection. Snell's law shows the relationship between the angle of incidence and the angle of refraction using the refractive indices and of the materials. By knowing  $n_1, n_2$ , and  $\theta_i$ , it's possible to calculate the angle of refraction  $\theta_t$ . This principle underlies the design of optical devices like lenses and beam splitters.

## Reflection and refraction



Light entering at a general angle will transmit (refract) and reflect according to Snell's law and the law of reflection.

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*In the example above:*

$\theta_i$  is the angle of incidence, the angle between the incoming light ray and the line perpendicular to the surface (the normal).

$\theta_r$  is the angle of reflection, the angle between the reflected light ray and the normal.

$\theta_t$  is the angle of refraction, the angle between the transmitted light ray (bending into the second medium) and the normal.

When light moves from a denser material (higher refractive index) to a less dense material (lower refractive index), such as from glass to air, an interesting phenomenon can occur: total internal reflection.

Total internal reflection is a cornerstone for technologies like:

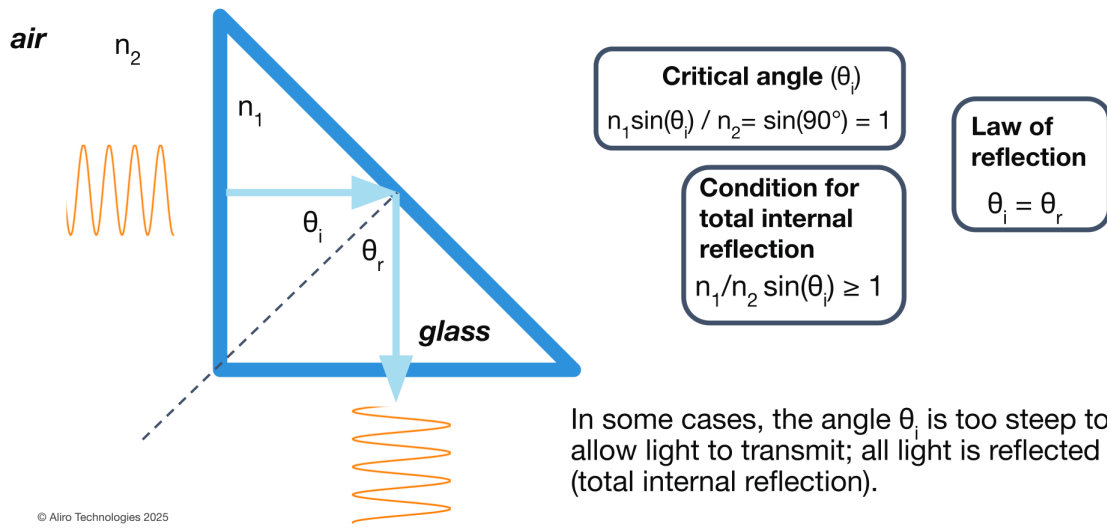
**Fiber Optic Cables.** Light signals are confined within fibers by repeated total internal reflection. This enables efficient and low loss long-distance data transmission.

**Optical Devices.** Beam splitters and waveguides use total internal reflection to manipulate light paths.

Total internal reflection occurs when the angle of incidence exceeds a critical angle, which is determined by the refractive indices of the two materials: in this case the light cannot refract into the less dense medium and instead all the light reflects back into the denser medium.

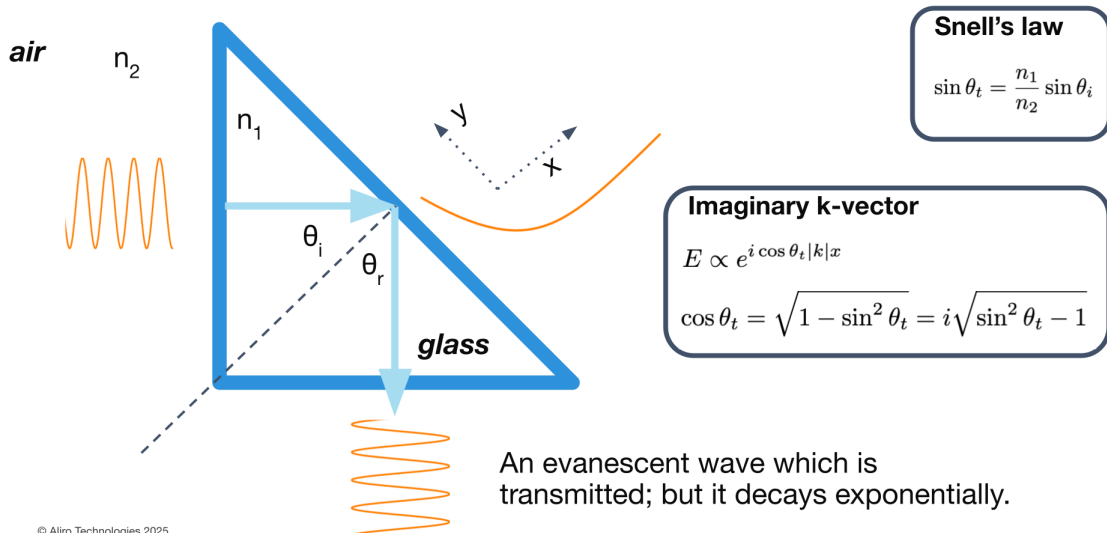
In cases where the angle  $\theta_i$ , the angle of incidence, is too steep to allow light to transmit, all light is reflected (total internal reflection).

## Total internal reflection



While total internal reflection prevents light from propagating into the less dense medium, a non-propagating electromagnetic field, known as an evanescent light wave, forms at the boundary. This wave decays exponentially with distance from the interface and does not carry energy into the second medium. However, it plays a crucial role in phenomena like frustrated total internal reflection, which is discussed in more detail below.

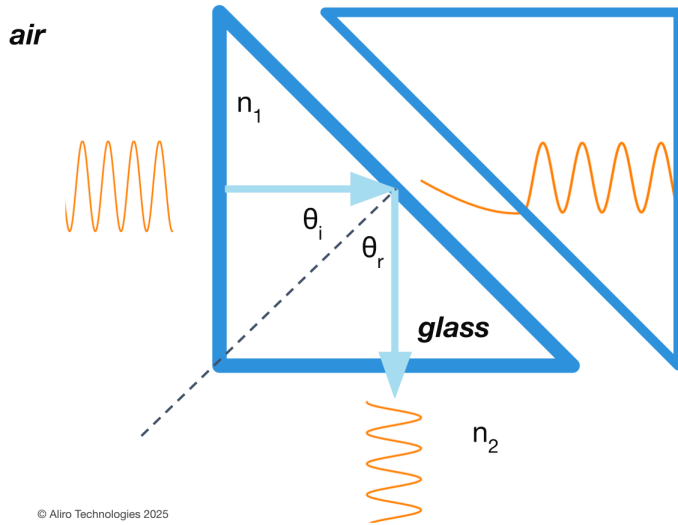
## Total internal reflection



Beam splitters can be used to create frustrated total internal reflection. Consider two glass prisms brought close together with a small gap filled by a material of lower refractive index (for example,

resin). If the gap width is comparable to the wavelength of light, the evanescent wave can penetrate the second prism, allowing partial transmission of light.

### Frustrated total internal reflection

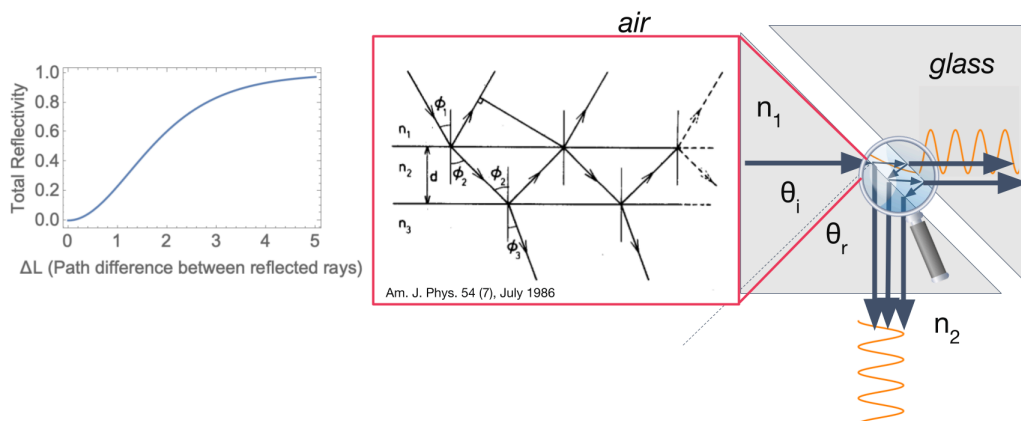


A small low index gap (order of wavelengths of light) between two higher index materials allows some light to transmit and some light to reflect.

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When the prisms are pressed together (with no gap between them), nearly all the light transmits due to maximal destructive interference effects for the reflected beam. As the gap between prisms increases, interference effects diminish the transmitted light, causing more light to reflect. Complete reflection can eventually occur, due to evanescent decay affecting the transmitted light when total internal reflection is not frustrated.

### Frustrated total internal reflection



#### Reflected electric field

$$E_r = r_{12}E_i + t_{12}e^{i\delta}t_{21}r_{23}E_i + t_{21}r_{21}r_{23}^2t_{12}e^{2i\delta}E_i + \dots$$

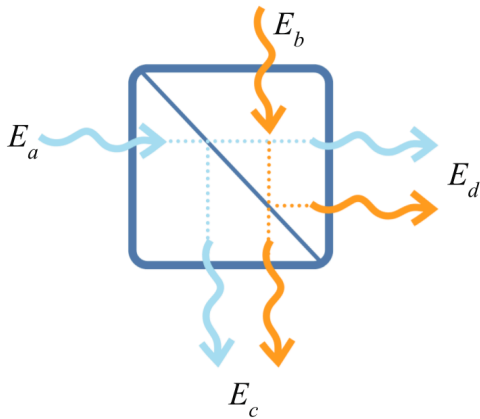
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This behavior clearly shows the law of energy conservation is maintained in optical systems. The ability to control the split between reflection and transmission makes beam splitters highly useful in optical systems. In fact, this is how a cube beam splitter works. Cube beam splitters are constructed from two glass triangular prisms glued together with a resin that has a lower refractive index, allowing precise manipulation of light transmission and reflection because of frustrated total internal reflection.

## Representing a Beam Splitter Mathematically

Mathematical models can be used to describe how incoming light interacts with the beam splitter. These models are used to understand, predict, and optimize their behavior in quantum systems, such as in a quantum network. A beam splitter can be represented mathematically as a matrix that operates on a vector containing the electric field amplitudes of the incoming light. This matrix describes how light entering the beam splitter is divided into transmitted and reflected light.



### <SLIDE 16 beam splitter image>

The input vector represents the electric field amplitudes of the incoming light:

$E_a$  is the electric field amplitude of light entering from the left.

$E_b$  is the electric field amplitude of light entering from the top.

The output vector represents the electric field amplitudes of the outgoing light:

$E_c$  The electric field amplitude of light exiting from the bottom.

$E_d$  The electric field amplitude of light exiting to the right.

The relationship between these vectors can be expressed as:

## How can we model beam splitters?

$$\begin{bmatrix} E_c \\ E_d \end{bmatrix} = \begin{bmatrix} r_{ac} & t_{bc} \\ t_{ad} & r_{bd} \end{bmatrix} \begin{bmatrix} E_a \\ E_b \end{bmatrix}$$

Electric field  
amplitude of  
exiting light

Beam splitter

Electric field  
amplitude of  
entering light

Here  $r$  and  $t$  represent the reflectivity and transmissivity coefficients, respectively.

These coefficients may be complex to account for any phase shifts introduced during reflection or transmission.

An important constraint when modeling beam splitters is the conservation of energy. The intensity of the incoming light must equal the intensity of the outgoing light. Light intensity is proportional to the square of the electric field amplitude.

## How can we model beam splitters?

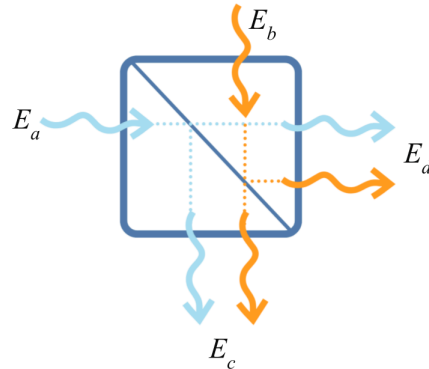


Conservation of energy:

$$|E_a|^2 + |E_b|^2 = |E_c|^2 + |E_d|^2$$

Definition of light intensity with respect to electric field:

$$A_i = |E_i|^2$$



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This relationship ensures that no energy is lost or created as light interacts with the beam splitter. The reflectivity and transmissivity coefficients  $r$  and  $t$  must also satisfy specific constraints.

## How can we model beam splitters?



$$\begin{bmatrix} E_c \\ E_d \end{bmatrix} = \begin{bmatrix} r_{ac} & t_{bc} \\ t_{ad} & r_{bd} \end{bmatrix} \begin{bmatrix} E_a \\ E_b \end{bmatrix}$$

Electric field amplitude of exiting light

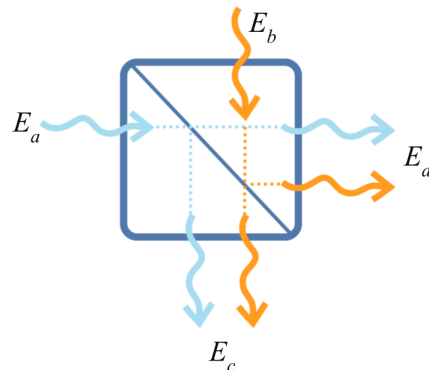
Beam splitter

Electric field amplitude of entering light

$r$  and  $t$  are in general complex, because a beam splitter can impart a phase

$$|r|^2 = 1 - T$$

$$|t|^2 = T$$



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These conditions link the coefficients to the physical properties of the beam splitter.

To account for phase changes that occur during reflection and transmission, the coefficients  $r$  and  $t$  are represented as complex numbers. After applying all these constraints, a refined matrix model for the beam splitter can be written described by four parameters:

- $\theta$ : mixing angle
- $\Phi_0$ : overall phase
- $\Phi_R$ : reflection phase
- $\Phi_T$ : transmission phase

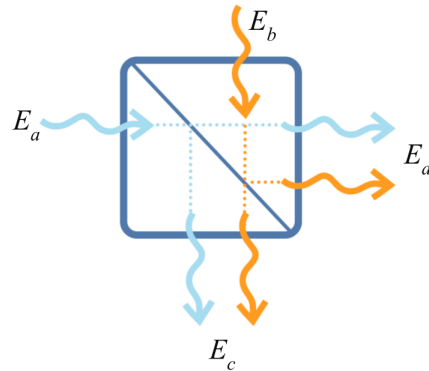
## How can we model beam splitters?



After applying constraints, the matrix for a beam splitter can be written:

$$\tau = e^{i\phi_0} \begin{bmatrix} \cos \theta e^{i\phi_R} & \sin \theta e^{-i\phi_T} \\ \sin \theta e^{i\phi_T} & -\cos \theta e^{-i\phi_R} \end{bmatrix}$$

- $\theta$ : mixing angle
- $\Phi_0$ : overall phase
- $\Phi_R$ : reflection phase
- $\Phi_T$ : transmission phase



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## Example: the 50/50 beam splitter

One of the most common types of beam splitter is the 50/50 beam splitter, which evenly splits an incoming light wave into two paths: 50% transmission and 50% reflection. These 50/50 beam splitters can be modeled mathematically based on the above mathematical principles.

A 50/50 beam splitter is characterized by a mixing angle of  $\theta = \pi/4$ . This results in a beam of light having an equal probability of being transmitted or reflected. There are two types of 50/50 beam splitter:

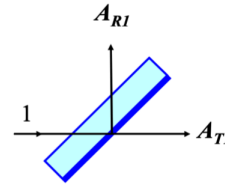
- Asymmetric phase, often implemented with half-silvered mirrors.
- Symmetric phase, often implemented with cube beam splitters.

## Transfer matrix examples

50:50 beam splitters

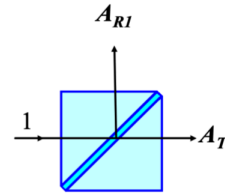
$$\tau = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{Asymmetric phase (half-silvered mirrors)}$$

$$(\theta = \pi/4, \phi_0 = \phi_T = \phi_R = 0)$$



$$\tau = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \quad \text{Symmetric phase (cube beam splitter)}$$

$$(\theta = \pi/4, \phi_0 = \pi/2, \phi_T = 0, \phi_R = -\pi/2)$$



F. Henault, Proc. SPIE 9570, (2015)

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To describe this process mathematically in asymmetric phase beam splitters, we use a transfer matrix, which relates the input and output electric fields.

In the case of asymmetric phase shift, all phase parameters ( $\phi_0$ ,  $\phi_T$ , and  $\phi_R$ ) are set to zero.

The phase that's imparted on the light is asymmetric and depends on which port of the beam splitter light is entering into. For example:

- Light entering from the left experiences no phase shift upon transmission or reflection.
- Light entering from the bottom transmits without phase shift, but reflects with a  $\pi$  phase shift (represented by the -1 in the matrix).

When light travels through a material with lower refractive index and reflects off a surface with a higher refractive index, it undergoes a phase shift of  $\pi$ .

Unlike asymmetric beam splitters, symmetric 50/50 beam splitters apply the same phase shift regardless of the input port. One of the best examples is the cube beam splitter, which maintains symmetry in phase application. This uniform phase application is particularly useful in quantum optics and interferometry, where phase consistency is critical.

In the case of symmetric phase shift, phase parameters are  $\phi_0 = \pi/2$ ,  $\phi_T = 0$ , and  $\phi_R = -\pi/2$ . The transfer matrix is then given by:  $\tau = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

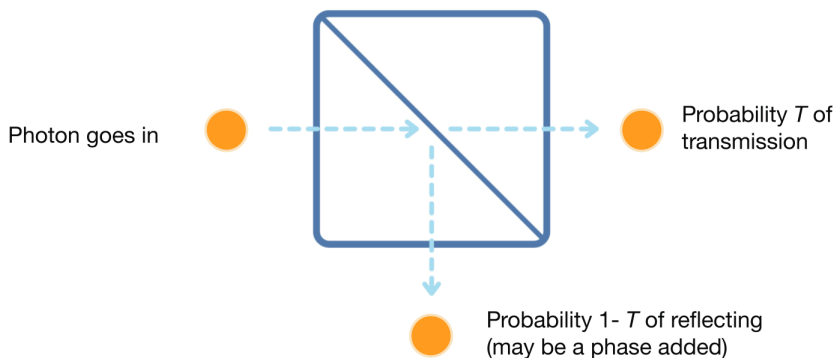
Light entering from any port experiences the exact same phase shift upon transmission or reflection.

## How beam splitters work (quantum optics)

In the quantum world, light is not a wave. Light is quantized, meaning it's made up of discrete particles. These particles are called photons. While a classical light wave can be partially transmitted and partially reflected by a beam splitter, a single photon must either be transmitted or reflected.

### What is a beam splitter? (Quantum picture)

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A beam splitter is like a passive switch for **photons** (quantum light).

Application: Quantum key exchange (BBM-92)

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Any photon entering a beam splitter has a probability of taking one path or the other, but the outcome is fundamentally uncertain: the photon is in superposition of both outcomes until measured. According to quantum mechanics, the behavior of photons in a beam splitter follows these probabilistic rules:

- The probability of transmission is given by  $T$ .
- The probability of reflection is  $1 - T$ .
- Phase shifts can be introduced upon transmission or reflection.

This property is particularly useful for quantum key exchange protocols like BBM92, which leverage this randomness for secure communication. The beam splitter essentially acts as a passive switch.

## Mathematical representation (quantum optics)

The quantum picture of beam splitters can still be described using transfer matrices. However, instead of dealing with electric field amplitudes, we now describe light in terms of photons, or qubits. The input state of a photon entering the beam splitter can be expressed as a quantum state vector.

The mathematical description of a beam splitter with respect to quantum physics uses the same transfer matrix as the classical optics, with some additional components.

For example: the asymmetric phase beam splitter transfer matrix:

$$\tau = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



## Beam splitter action on a photon

A beam splitter can put a single photon (aka a qubit) into a [superposition state](#).

$$\begin{aligned}
 U_{BS} |0\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
 \end{aligned}$$

Transfer matrix for beam splitter
Qubit initial state

Qubit final state

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This matrix is multiplied by a vector, which is not the electric field of the light wave but rather describes the qubit passing through the beam splitter. The components in this vector describe the probabilities that a qubit is entering into one or the other ports of this beam splitter. The transfer matrix expression,

$$\tau = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

multiplied by the expression for a qubit  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  that enters into one beam splitter port, equals

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

which is the superposition state for a qubit. This means that this qubit has a 50% chance of having transmitted and a 50% chance of having reflected. There is no way to know if it's transmitted or reflected until it is actually measured.

This expression,  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$  is also a Hadamard gate. This is an important gate for quantum computing applications for putting qubits into superposition states. This is how a 50/50 beam splitter can act as a Hadamard gate for a photonic qubit traveling on one or another path, an important application for quantum computing.

## Generalizing the model

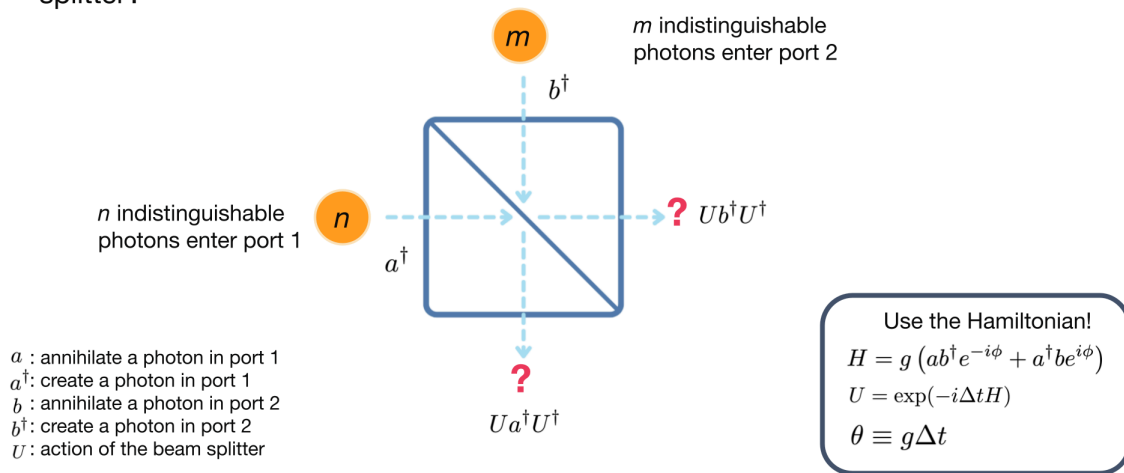
While we have explored various ways to model beam splitters and their interaction with individual photons or light waves, we have yet to consider what happens when multiple indistinguishable photons enter a beam splitter simultaneously. Specifically, if  $m$  indistinguishable photon enters from

one port and n indistinguishable photon enters from another, how do they behave upon encountering the beam splitter? Understanding this is essential for real-world applications, where multi-photon interactions are common and crucial to quantum technologies.



## Generalizing the model

What happens when more than one indistinguishable photon enters a beam splitter?



To model the behavior of multiple indistinguishable photons entering a beam splitter, we need to analyze how the creation and annihilation operators transform under the action of the beam splitter's unitary evolution. These operators, denoted as  $a$ ,  $a^\dagger$ ,  $b$ , and  $b^\dagger$ , describe the quantum states of photons entering the beam splitter from two different ports:

- $a$  (annihilation operator): Removes a photon from input port 1.
- $a^\dagger$  (creation operator): Adds a photon to input port 1.
- $b$  (annihilation operator): Removes a photon from input port 2.
- $b^\dagger$  (creation operator): Adds a photon to input port 2.

An arbitrary quantum state consisting of photons can be created by starting with the vacuum state (the state with no photons) and adding creation operators to that state. A beam splitter's action on photon operators can be represented by matrix  $U$ , a unitary transformation. This transformation can either:

- Act on the quantum state directly.
- Transform the creation and annihilation operators associated with the state.

To do this, we introduce an expression as an initial guess for how the beam splitter modifies photon states, then verify that this expression reproduces expected results in the single-photon case using transfer matrices. This approach allows us to extend our findings to multiple-photon scenarios.

The unitary transformation is parameterized by a Hamiltonian, where:  
 $g$  is a coupling term.

$\phi$  is a phase factor.

$\theta = g\Delta t$  represents the mixing angle for the beam splitter transformation.

Solving for the unitary evolution of the operators, we derive expressions that match the classical transfer matrix formalism when certain values of  $\theta$ ,  $\phi$ , and phase shifts are chosen. This confirms the validity of the operator approach in describing beam splitter behavior in the quantum regime.

To complete the derivation, we solve for the transformed creation operators  $U a^\dagger U^\dagger$  and  $U b^\dagger U^\dagger$ , which describe how photon states evolve through the beam splitter.

To determine how the unitary operator  $U$ , representing the beam splitter's action, interacts with and affects the operators  $a^\dagger$  and  $b^\dagger$ , solve for:

$$U a^\dagger U^\dagger = \cos(\theta) a^\dagger - ie^{-i\phi} \sin(\theta) b^\dagger$$

$$U b^\dagger U^\dagger = \cos(\theta) b^\dagger - ie^{-i\phi} \sin(\theta) a^\dagger$$

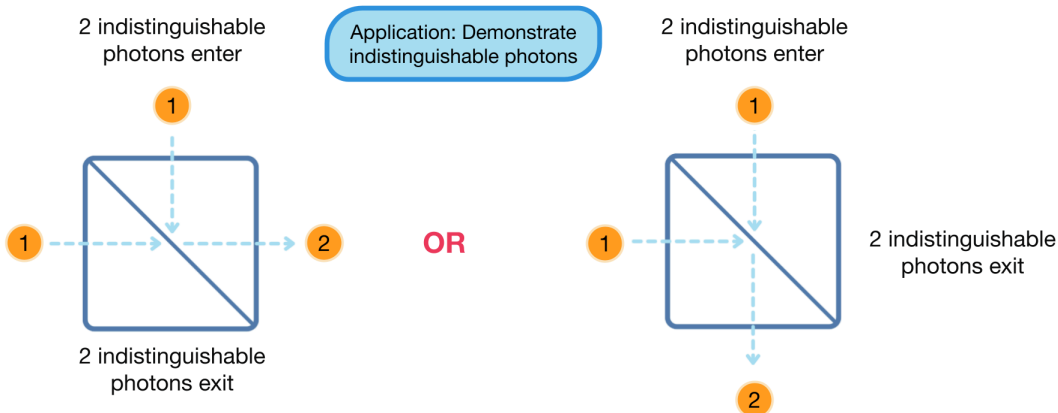
In this process, the Baker-Campbell-Hausdorff theorem can be a useful mathematical tool for handling the exponential operators involved.

## The Hong-Ou-Mandel effect

The Hong-Ou-Mandel (HOM) effect is a quantum interference phenomenon that can be used to test for photon indistinguishability. The effect occurs when two identical photons enter a 50/50 beam splitter from separate input ports. Instead of randomly exiting through either of the two output ports, the photons always bunch together, exiting through the same port. This occurs experimentally, and also can be derived through the operator formalism for the beam splitter discussed previously.

### Hong-Ou-Mandel effect

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**Two identical photons exiting a 50:50 beam splitter will bunch.**



## Hong-Ou-Mandel effect

The math:

$$\begin{aligned}
 U |1, 1\rangle &= U a^\dagger b^\dagger |0, 0\rangle \\
 &= (U a^\dagger U^\dagger)(U b^\dagger U^\dagger) |0, 0\rangle \\
 &= \frac{1}{2} (a^\dagger - i e^{-i\phi} b^\dagger) (b^\dagger - i e^{i\phi} a^\dagger) |0, 0\rangle \\
 &= \frac{1}{2} (a^\dagger b^\dagger - b^\dagger a^\dagger - i e^{i\phi} a^{\dagger 2} - i e^{-i\phi} b^{\dagger 2}) |0, 0\rangle \\
 &= \frac{1}{\sqrt{2}} (e^{i\phi} |2, 0\rangle + e^{-i\phi} |0, 2\rangle).
 \end{aligned}$$

**Two identical photons exiting a 50:50 beam splitter will bunch.**

Working through the quantum mechanics, one can predict that the final state of the photons after the beam splitter is a superposition of them exiting in the same direction.

### Bell state measurements

Another critical application of beam splitters in quantum optics is Bell state measurements (BSMs), which are essential for quantum entanglement distribution and quantum teleportation in quantum networks.

Bell states are the four maximally entangled quantum states of a two-qubit system. These states ensure that measurements of one qubit are maximally correlated with the results of measurements on the other qubit. The four Bell states are:

- $\Psi^+$  (Psi plus)
- $\Psi^-$  (Psi minus)
- $\Phi^+$  (Phi plus)
- $\Phi^-$  (Phi minus)

These states are crucial in quantum information because they enable the transmission of entanglement over long distances.

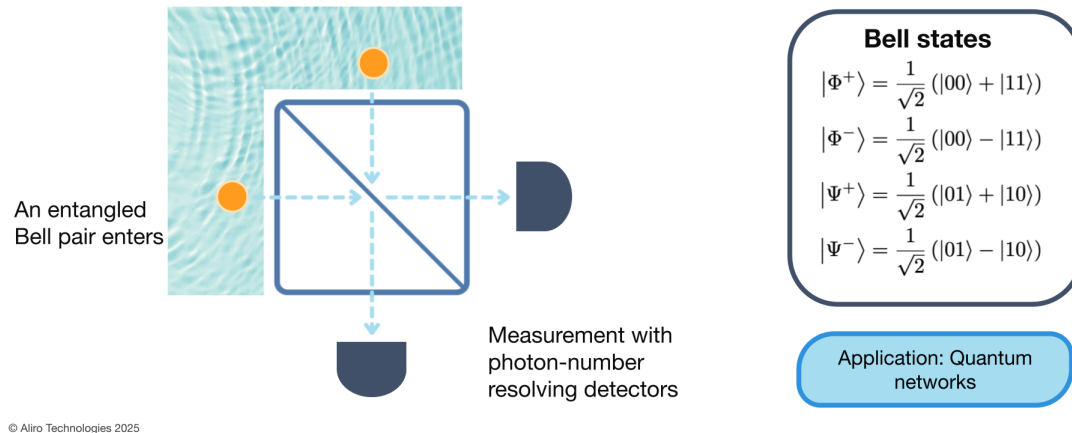
One way a Bell state measurement can be implemented is by using a beam splitter and two photon-number-resolving detectors at its output ports. The process works as follows:

- An entangled Bell pair is sent into the beam splitter.
- Using the previously derived unitary transformations for the beam splitter, we can predict the expected photon distributions at the detectors.
- Different Bell states produce distinct click patterns on the detectors:

- The  $\Psi^+$  state results in a single detector click.
- The  $\Psi^-$  state results in a single detector click at the opposite detector.
- The  $\Phi^+$  state results in either two clicks at one detector or no clicks at all.
- The  $\Phi^-$  state produces the same pattern as the  $\Phi^+$  state.

## Bell state measurements

A 50:50 beam splitter with the appropriate phase can be used to perform a Bell state measurement.



While a 50/50 beam splitter can distinguish between  $\Psi^+$  and  $\Psi^-$ , it cannot differentiate between  $\Phi^+$  and  $\Phi^-$ . This means that any Bell state measurement relying solely on linear optical elements (like beam splitters) has an intrinsic success rate of 50%. This limitation is fundamental to linear optical quantum computing and requires additional methods to improve measurement fidelity.

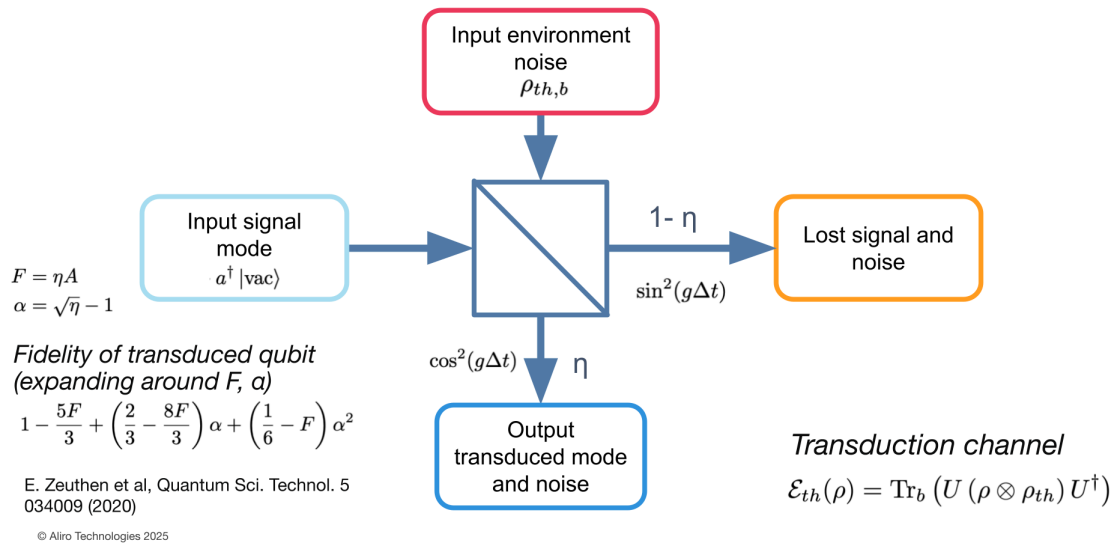
## Modeling quantum frequency transduction

Quantum frequency transduction is essential for quantum networks, allowing photons or qubits to be converted between different energy levels. This conversion is critical because different photon energies are better suited for different tasks, such as long-distance transmission or quantum computing operations.

A particularly interesting case of frequency transduction is red-detuned transduction: the Hamiltonian for red-detuned transduction is the same as the beam splitter Hamiltonian. Understanding that red-detuned transduction follows a beam splitter Hamiltonian provides a powerful insight: beam splitter models are highly generalizable and can describe various quantum mechanical interactions beyond their traditional optical applications.

Recognizing the connection between frequency transduction and beam splitters allows us to use well-established beam splitter mathematics to analyze quantum transduction fidelity.

## Modeling transduction with noise



Specifically, we can compute the fidelity of a transduced qubit by considering:

$\eta$  (Eta): The efficiency of transduction, representing the probability that a qubit is successfully converted.

$F$ : The environmental noise parameter, corresponding to the average number of noisy thermal photons present in the system.

To analyze how these factors influence transduction, we model the transduction as a two-port beam splitter, where the signal photons will enter one port and noisy thermal photons enter another port. The challenge here is that we may not be able to directly measure the quantum state of the photons in the environmental noise. However, instead of needing a complete description of each noise photon's quantum state, we can perform a partial trace over the environmental noise. This allows us to average out the unwanted thermal photons and focus solely on their impact on the transduced qubit.

By following this approach, we can derive an expression for the fidelity of the transduced qubit based on noise and efficiency parameters.

One particularly interesting scenario is where the environmental temperature  $T = 0$ . In this scenario:

- No thermal photons interfere with the transduction process.
- The noise term  $F$  is set to zero.
- The Kraus operators simplify and transform into amplitude damping Kraus operators, which describe a process where there is no added noise, only a probability that the qubit successfully transduces or remains unchanged.

This highlights how the beam splitter Hamiltonian provides a clear and flexible framework for analyzing complex quantum interactions in various domains, from photonic networks to qubit frequency conversion.

## Conclusion

Beam splitters are fundamental tools in quantum optics, serving as essential components for manipulating and measuring light at both the classical and quantum levels. They provide the foundation for understanding optical interference, quantum superposition, and entanglement, as demonstrated by effects like the Hong-Ou-Mandel (HOM) effect, where indistinguishable photons always exit together from the same port of a beam splitter. These devices also play a crucial role in Bell state measurements, which are key to entanglement-based quantum communication protocols.

Beyond their traditional role in optical systems, beam splitter formalisms extend to multi-photon interference and even non-optical applications such as quantum frequency transduction. In quantum networks, frequency transduction allows qubits to be converted between different energy levels, enabling compatibility between diverse quantum systems. Remarkably, red-detuned quantum frequency transduction follows the same mathematical structure as a beam splitter Hamiltonian, demonstrating the generalizability of these models. By leveraging this framework, researchers can optimize quantum state fidelity in the presence of noise, a critical requirement for building scalable quantum communication networks.

The generality of beam splitter models underscores their significance across various quantum technologies, providing a robust mathematical framework for analyzing photon interactions, multi-photon interference, and quantum transduction. As quantum systems evolve, these formalisms will remain indispensable tools for advancing quantum computing, secure communication, and photonic quantum technologies. Future research will continue to refine these models, optimizing their application in increasingly complex quantum architectures.

## The Future is Entanglement-based Quantum Networks

As we look to the future of secure communications, entanglement-based quantum networks are at the forefront. Building these networks requires specialized software and hardware, such as beam splitters.

Entanglement-based quantum networks are being built today by a variety of organizations for a variety of use cases – benefiting organizations internally, as well as providing great value to an organization's customers. Telecommunications companies, national research labs, and systems integrators are just a few examples of the organizations Aliro is helping to leverage the capabilities of quantum secure communications.

Aliro is uniquely positioned to help you build your quantum network. The steps you can take to ensure your organization is meeting the challenges and leveraging the benefits of the quantum revolution are part of a clear, unified solution already at work in networks like the EPB Quantum Network<sup>SM</sup> powered by Qubitekk in Chattanooga, Tennessee.

AliroNet<sup>TM</sup>, the world's first full-stack entanglement-based network solution, consists of the software and services necessary to ensure customers will fully meet their advanced secure networking goals.

Each component within AliroNet™ is built from the ground up to be compatible and optimal with entanglement-based networks of any scale and architecture. AliroNet™ is used to simulate, design, run, and manage quantum networks as well as test, verify, and optimize quantum hardware for network performance. AliroNet™ leverages the expertise of Aliro personnel in order to ensure that customers get the most value out of the software and their investment.

Depending on where customers are in their quantum networking journeys, AliroNet™ is available in three modes that create a clear path toward building full-scale entanglement-based secure networks: (1) Emulation Mode, for emulating, designing, and validating entanglement-based quantum networks, (2) Pilot Mode for implementing a small-scale entanglement-based quantum network testbed, and (3) Deployment Mode for scaling entanglement-based quantum networks and integrating end-to-end applications. AliroNet™ has been developed by a team of world-class experts.

To get started on your Quantum Networking journey, reach out to the Aliro team for additional information on how AliroNet™ can enable secure communications.

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